Analytical Solution and Parametric Study of Free Transverse Vibration of Nano Scale Beam

A THESIS

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By

Lipika Parida

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Prof. S. Chakraverty

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DEPARTMENT OF MATHEMATICS

NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA-769008

ODISHA, INDIA

NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **"Analytical Solution and Parametric Study of Free Transverse Vibration of Nano Scale Beam"** in partial fulfillment of the requirement for the award of the degree of Master of Science, submitted in the Department of Mathematics, National Institute of Technology, Rourkela is an authentic record of my own work carried out under the supervision of Dr. S. Chakraverty.

The matter embodied in this thesis has not been submitted by me for the award of any other degree .

Date: (LIPIKA PARIDA)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

Dr. S. Chakraverty

Professor, Department of Mathematics

National Institute of Technology

Rourkela-769008

Odisha, India

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Thanks to all my classmates and friends for their support and encourage.

Thanks too, to all of my family members who have all supported me on my journey.

Finally, thanks to all my good wisher. It has been a pleasure to work on mathematics in National Institute of Technology, Rourkela.

Abstract

In this project free vibration of the nano scale beam based on the non-local continuum models has been studied. Corresponding equation of motion is of fourth order ordinary differential equation. The boundary conditions have been taken as simply supported boundary. Frequency parameters of the said problem have been obtained by solving the fourth order differential equation. Various parametric studies are done and corresponding Tables and Plots are incorporated to understand the nanotechnology aspects related to non-local and local nano beams. The results in terms of parametric investigations obtained may give an idea about design of nano beams.

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Chapter -1

Introduction

In the present time, the non-local elasticity is used to investigate the free transverse vibration of nano-to-micro scale beam. The frequency equation is analytically formulated into eigen value problem. For the beams with micrometer scale length the impact of non-local effect on the natural frequencies and vibrating modes is negligible whereas the effect of non-local becomes important in case of the beam with nanometer scale length.

Due to the rapidly developing nanotechnology, more microeletromechanical devices can be brought up. These devices can be of nanometer scale. The increased requirements in applications have drawn tremendous attention to the mechanical properties of the nano scale devices. The two approaches available for such investigations are the classical continuum mechanics and the atomic or molecular models. But they have some drawbacks. The application of classical continuum mechanics to nanoscale devices is not confirmed [2, 3, 12]. On the other hand atomic or molecular model is a powerful tool to investigate the properties of nanoscale devices. Therefore the non-local continuum mechanics [4, 5, 7] which is the improvement of classical continuum mechanics came into picture by counting the impact of the small scale effect. It helps us to deal with the tiny devices [10]. The small scale effects are key components in microelectromechanical systems, such as cantilever actuators, nanotubes, nanowires. Usually, the classical Bernoulli/Euler beam model is employed to describe the vibration of these nano scale beam structures. [8, 11, 13, 14, 15, 16]. The non-local elasticity theory has been developed to investigate the bending moment of the nano scale beams [10] and the free transverse vibrations of carbon nanotubes [17].

So, here we focus on the free vibration of the general nano scale beams based on the non-local continuum models [9]. The problem in particular to vibration analysis has not been investigated much. Few authors have studied the same and those are given in the Reference list. The list also contains few other papers which are related to the subject of this study.

Chapter -2

Modeling and Problem Formulation

The uniaxial Hooke's Law for the non-local elasticity is expressed as [6, 10, 17]

$$\sigma - (e_0 a)^2 \frac{\partial^2 \sigma}{\partial x^2} = E\varepsilon \qquad (1)$$

where, σ = the axial stress

ε = the axial strain

E = the Young's modulus

a = the internal characteristic length

e_0 = the constant which is determined experimentally

For the Transverse vibration of beam, the equilibrium conditions are[1, 10, 17]

$$S = \frac{\partial M}{\partial x} \qquad (2)$$

$$\frac{\partial S}{\partial x} = -p + \rho A \frac{\partial^2 W}{\partial t^2} \qquad (3)$$

where, S = the shear force

M = the bending moment

p = the mass density

A = the cross-sectional area

Combining equations (2) and (3), we get

$$\frac{\partial^2 M}{\partial x^2} = -p + \rho A \frac{\partial^2 W}{\partial t^2} \qquad (4)$$

Multiplying y on both sides of equation (1) and integrating over the cross-sectional area of the beam at the point x, we get

$$\int_A \sigma y dA - (e_0 a)^2 \int_A y \frac{\partial^2 \sigma}{\partial x^2} dA = \int_A E y \varepsilon dA \tag{5}$$

From definition of the bending moment, we have

$$\mathrm{M} = \int_A \sigma y dA \tag{6}$$

The relation between strain and curvature for the small deflection Bernoulli/Euler beam is

$$\varepsilon = -y \frac{\partial^2 W}{\partial x^2} \tag{7}$$

where, W is the beam's transverse displacement.

Substituting equations (6) and (7) into equation (5), we get for the uniform beam

$$M - (e_0 a)^2 \frac{\partial^2 M}{\partial x^2} = -EI \frac{\partial^2 W}{\partial x^2} \tag{8}$$

where I is the moment of inertia.

Differentiating twice both sides of the above equation with respect to the variable x, we obtain

$$\frac{\partial^2 M}{\partial x^2} - (e_0 a)^2 \frac{\partial^4 M}{\partial x^4} = -\frac{\partial}{\partial x^2} \left(EI \frac{\partial^2 W}{\partial x^2} \right) \tag{9}$$

Now substituting equation (4) into equation (9), we get the equation of beams based on the non-local continuum elasticity [17],

$$EI \frac{\partial^4 W}{\partial t^4} + \rho A \frac{\partial^2 W}{\partial t^2} - (e_0 a)^2 \left(\rho A \frac{\partial^4 W}{\partial^2 x \partial^2 t} - \frac{\partial^2 p}{\partial x^2} \right) - p = 0 \tag{10}$$

By setting ξ=x/L in equation (1), we get the following beam equation as

$$\left(\frac{EI}{L^4}\right)\frac{\partial^4 W}{\partial t^4}+\rho A\frac{\partial^2 W}{\partial t^2}-\left(\frac{e_0 a}{L}\right)^2\left(\rho A\frac{\partial^4 W}{\partial^2 x\partial^2 t}-\frac{\partial^2 p}{\partial x^2}\right)-p=0 \tag{11}$$

(where, we have written x for ξ)

If the co-efficient of the third term in the bracket of equation (11) becomes zero then it can be written as

$$\left(\frac{EI}{L^4}\right)\frac{\partial^4 W}{\partial t^4}+\rho A\frac{\partial^2 W}{\partial t^2}-p=0 \tag{12}$$

This is the Local Euler beam equation under forced vibration.

For free vibration of beam, p is equal to zero and for transverse vibration, we may assume the solution of equation (11) as $W\,(x,t) = w(x)e^{i\omega t}$ and accordingly we get

$$\frac{\partial^4 w(x)}{\partial x^4}-\lambda\left(w(x)-\mu\frac{\partial^2 w(x)}{\partial x^2}\right)=0 \tag{13}$$

where,

$$\lambda=\frac{\rho A L^4\omega^2}{EI} \quad \text{and} \quad \mu=\frac{(e_0 a)^2}{L^2} \tag{14}$$

Here ω is the natural frequency of the nano beam and equation (13) is the Euler non-local beam equation for nano-scale.

If μ=0, then equation (13) becomes

$$\frac{\partial^4 w(x)}{\partial x^4}-\lambda w(x)=0 \tag{15}$$

It will give us the traditional classical Euler beam equation.

Chapter -3

Solution

Equation (13) may easily be written as $D^4+\mu\lambda D^2-\lambda=0$, which gives

$$D=\pm\sqrt{\frac{-\lambda\mu}{2}\pm\frac{\lambda\mu}{2}\sqrt{1+\frac{4}{\lambda\mu^2}}}$$

where,

$$\beta_1=\pm\sqrt{\frac{-\lambda\mu}{2}+\frac{\lambda\mu}{2}\sqrt{1+\frac{4}{\lambda\mu^2}}} \tag{16}$$

$$\beta_2=\pm\sqrt{\frac{-\lambda\mu}{2}-\frac{\lambda\mu}{2}\sqrt{1+\frac{4}{\lambda\mu^2}}} \tag{17}$$

Now the general solution is

$$w(x)=C_1e^{\beta_1 x}+C_2e^{-\beta_1 x}+C_3e^{\beta_2 x}+C_4e^{-\beta_2 x}$$

This can be written as

$$w(x)=A\cosh(\beta_1 x)+B\sinh(\beta_1 x)+C\cos(\beta_2 x)+D\sin(\beta_2 x) \tag{18}$$

The unknown coefficients *A*,*B*,*C* and *D* are obtained here by application of the classical simply-supported boundary conditions,

$$w(0)=0$$
$$w''(0)=0$$
$$w(1)=0$$
$$w''(1)=0$$

Substitution of equation (18) into the above boundary conditions yields a linear system of equations, whose determinant forms the system characteristics equations.

The solution of the characteristics equations gives the value of the constants *A*, *B*, *C* and *D* as $A=C=0$, $B\neq 0$, $D\neq 0$, $\beta_1=in\pi$ and $\beta_2=n\pi$.

Putting β_1 in equation (16), we get

$$\lambda=\frac{n^4\pi^4}{1+n^2\pi^2\mu}$$

Now substituting in equation (14), we get the natural frequency as

$$\omega_n=\frac{1}{L^2}\sqrt{\frac{EI}{\rho A}}\frac{n^2\pi^2}{\sqrt{1+\mu n^2\pi^2}} \tag{19}$$

where, ω_n is the natural frequency of the non-local Euler beam and n=1, 2, 3,…..

Special Case: When $\mu=0$, Equation (13) can be written as $D^4-\lambda=0$, which gives

$$D=\pm\sqrt{\pm\sqrt{\lambda}}$$

where,

$$\beta_1=\pm\sqrt{+\sqrt{\lambda}} \tag{20}$$

$$\beta_2=\pm\sqrt{-\sqrt{\lambda}} \tag{21}$$

Now the general solution is

$$w(x)=C_1e^{\beta_1x}+C_2e^{-\beta_1x}+C_3e^{\beta_2x}+C_4e^{-\beta_2x}$$

which can be written as

$$w(x)=A\cosh(\beta_1x)+B\sinh(\beta_1x)+C\cos(\beta_2x)+D\sin(\beta_2x) \tag{22}$$

The unknown coefficients *A*,*B*,*C* and *D* are again obtained by application of the classical simply-supported boundary conditions,

$$w(0) = 0$$
$$w''(0) = 0$$
$$w(1) = 0$$
$$w''(1) = 0$$

Similarly, substitution of equation (22) into the above boundary conditions yields a linear system of equations, whose determinant forms the system characteristics equations.

The solution of the characteristics equations gives the value of the constants *A*, *B*, *C* and *D* as $A=C=0$, $B\neq0$, $D\neq0$, $\beta_1=in\pi$ and $\beta_2=n\pi$.

Similarly Putting β_1 or β_2 in equation (20) or (21) respectively, we get the value of λ as $\lambda=n^4\pi^4$, and substituting in equation (14), we get the natural frequency as.

$$\omega = \frac{1}{L^2}\sqrt{\frac{EI}{\rho A}}n^2\pi^2 \tag{23}$$

It is interesting to note that, ω is the natural frequency of the local Euler beam.

We may also get this directly from equation (19) by setting $\mu=0$.

The impact of non-local effect on the natural frequencies of beams is represented by the ratio[9],

$$\varpi = \frac{\omega - \omega_n}{\omega} \tag{24}$$

Chapter -4

Results and Discussion

Above problem has been solved for various values of the parameters involved. But only few of them are reported. In Table1 we incorporate the first ten natural frequencies of the local and non-local simply supported beam with length L=60a for different values of e_0. As e_0 increases, the value of natural frequency decreases. Corresponding plots of natural frequencies for n=1 to 5 are shown in Figures 1 to 5 respectively. In Tables 2 and 3 we incorporate the impact of natural frequencies of simply supported beam with length L=60a and L=80a for different values of e_0 respectively. The impact of non-local effect on the natural frequency increases with increase in e_0. Corresponding plots of natural frequencies [equation (24)] for n=1 to 5 are depicted in Figures 6 to 10 and 11 to 15 respectively. Again in Tables 4 and 5 we incorporate the influence of natural frequencies of simply supported beam with different lengths for e_0=0.2 and e_0=0.82 respectively.

Table 1. The first ten natural frequencies of the simply supported local and non-local beam with length L=60a for different values of e_0.

n	ω_n (natural frequencies of the non-local beam)					ω (natural frequencies of the local beam)
	e_0=0.1	e_0=0.2	e_0=0.3	e_0=0.4	e_0=0.82	
1	0.0949234585	0.0949195553	0.0949130510	0.0949039472	0.0948373872	0.09492475967
2	0.3796782211	0.3796157888	0.3795118033	0.3793663671	0.3783068415	0.3796990387
3	0.8542174587	0.8539015575	0.8533758333	0.8526414481	0.8473228672	0.8543228371
4	1.518463155	0.517465469	1.515807019	1.513494298	1.496881489	1.518796155
5	2.372306155	2.369872645	2.365833385	2.360812962	2.320256027	2.373118992
6	3.41560623	3.410565793	3.402214425	3.39062491	3.309248863	3.417291348
7	4.648192168	4.63886656	4.62344784	4.602117659	4.454480482	4.651313224
8	6.069861874	6.053977192	6.027777617	5.991663146	5.745696027	6.075184619
9	7.6803825	7.654982457	7.613203361	7.555845375	7.172070726	7.688905534
10	9.479490581	9.440851849	9.377490777	9.290890833	8.722498229	9.492475967

Table 2. The impact of non-local effect on the natural frequencies [equation (24)] of the simply supported beam with length L=60a for different e_0.

N	e0=0.1		e0=0.2	e0=0.3		e0=0.4	e0=0.82	
	[1]	Present		[1]	Present		[1]	Present
1	1.3708×10^{-5}	1.37075×10^{-5}	5.48267×10^{-5}	1.2335×10^{-4}	1.23347×10^{-4}	2.19252×10^{-4}	9.2044×10^{-4}	9.20439×10^{-4}
2	5.4827×10^{-5}	5.48266×10^{-5}	2.19252×10^{-4}	4.9312×10^{-4}	4.93115×10^{-4}	8.76145×10^{-4}	3.6666×10^{-3}	3.66658×10^{-3}
3	1.2335×10^{-4}	1.23347×10^{-4}	4.93115×10^{-4}	1.1085×10^{-3}	1.10848×10^{-3}	1.96810×10^{-3}	8.1936×10^{-3}	8.19359×10^{-3}
4		2.19252×10^{-4}	8.76145×10^{-4}		1.96810×10^{-3}	3.49083×10^{-3}		1.44290×10^{-2}
5		3.42519×10^{-4}	1.36797×10^{-3}		3.07006×10^{-3}	5.43842×10^{-3}		2.22757×10^{-2}
6		4.93115×10^{-4}	1.96810×10^{-3}		4.41195×10^{-3}	7.80338×10^{-3}		3.16164×10^{-2}
7		6.71005×10^{-4}	2.67595×10^{-3}		5.99086×10^{-3}	1.05767×10^{-2}		4.23177×10^{-2}
8		8.76145×10^{-4}	3.49083×10^{-3}		7.80338×10^{-3}	0.01375×10^{-2}		5.42352×10^{-2}
9		1.10848×10^{-3}	4.41195×10^{-3}		9.84564×10^{-3}	0.01731×10^{-2}		6.74183×10^{-2}
10		1.36797×10^{-3}	5.43842×10^{-3}		1.21133×10^{-2}	0.02124×10^{-2}		8.11145×10^{-2}

Table 3. The impact of non-local effect on the natural frequencies [equation (24)] of the simply supported beam with length L=80a for different values of e_0.

n	e_0=0.1	e_0=0.2	e_0=0.3	e_0=0.4	e_0=0.82	
					[1]	Present
1	7.71054×10^{-6}	3.08410×10^{-5}	6.93884×10^{-5}	1.23347×10^{-4}	5.1806×10^{-4}	5.18059×10^{-4}
2	3.08411×10^{-5}	1.23347×10^{-4}	2.77467×10^{-4}	4.93115×10^{-4}	2.0674×10^{-3}	2.06742×10^{-3}
3	6.93884×10^{-5}	2.77467×10^{-4}	6.23977×10^{-4}	1.10848×10^{-3}	4.6338×10^{-3}	4.63376×10^{-3}
4	1.23347×10^{-4}	4.93115×10^{-4}	1.10848×10^{-3}	1.96810×10^{-3}		8.19359×10^{-3}
5	1.92710×10^{-4}	7.70172×10^{-4}	1.73039×10^{-3}	3.07006×10^{-3}		1.27149×10^{-2}
6	2.77467×10^{-4}	1.10848×10^{-3}	2.48892×10^{-3}	4.41195×10^{-3}		1.81579×10^{-2}
7	3.77607×10^{-4}	1.50787×10^{-3}	3.38314×10^{-3}	5.99086×10^{-3}		2.44759×10^{-2}
8	4.93115×10^{-4}	1.96810×10^{-3}	4.41195×10^{-3}	7.80338×10^{-3}		3.16164×10^{-2}
9	6.23976×10^{-4}	2.48892×10^{-3}	5.57410×10^{-3}	9.84564×10^{-3}		3.95226×10^{-2}
10	7.70172×10^{-4}	3.07006×10^{-3}	6.86815×10^{-3}	1.21133×10^{-2}		4.81337×10^{-2}

Table 4. The impact of non-local effect on the natural frequencies [equation (24)] of the simply supported beam with e_0=0.2 for different L/a.

N	L/a=100	L/a=70	L/a=50	L/a=30	L/a=10
1	0.094923	0.094921	0.094917	0.094904	0.094738
2	0.379669	0.379638	0.379579	0.379366	0.376736
3	0.854171	0.854013	0.853716	0.852641	0.839538
4	1.518317	1.517818	1.516881	1.513494	1.472987
5	2.371949	2.370733	2.368448	2.360213	2.264022
6	3.414866	3.412346	3.407619	3.390624	3.197612
7	4.646821	4.642159	4.633422	4.602118	4.257695
8	6.067524	6.059582	6.044716	5.991663	5.428034
9	7.676641	7.663939	7.640198	7.555845	6.692903
10	9.473794	9.454466	9.418403	9.290891	8.037593

Table 5. The impact of non-local effect on the natural frequencies [equation (24)] of the simply supported beam with e_0=0.82 for different L/a.

N	L/a=100	L/a=70	L/a=50	L/a=30	L/a=10
1	0.094893	0.094861	0.094799	0.094577	0.091923
2	0.379196	0.378675	0.377699	0.374220	0.337533
3	0.851783	0.849163	0.844297	0.827312	0.675979
4	1.510797	1.502603	1.487535	1.436424	1.057730
5	2.353675	2.333936	2.298089	2.180625	1.455304
6	3.377188	3.336911	3.264851	3.037799	1.856271
7	4.577483	4.504255	4.375448	3.986547	2.255741
8	5.950140	5.827858	5.616775	5.007465	2.652124
9	7.490225	7.298964	6.975468	6.083808	3.045160
10	9.192358	8.908372	8.438330	7.201674	3.435084

Plots:-

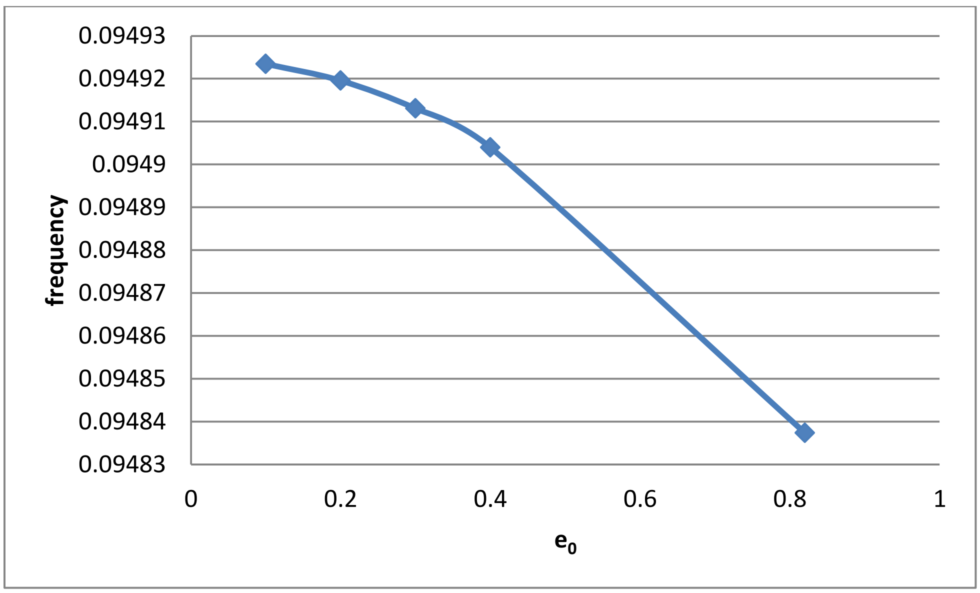

Fig 1: Effect of first frequency of non-local beam on various values of e_0.

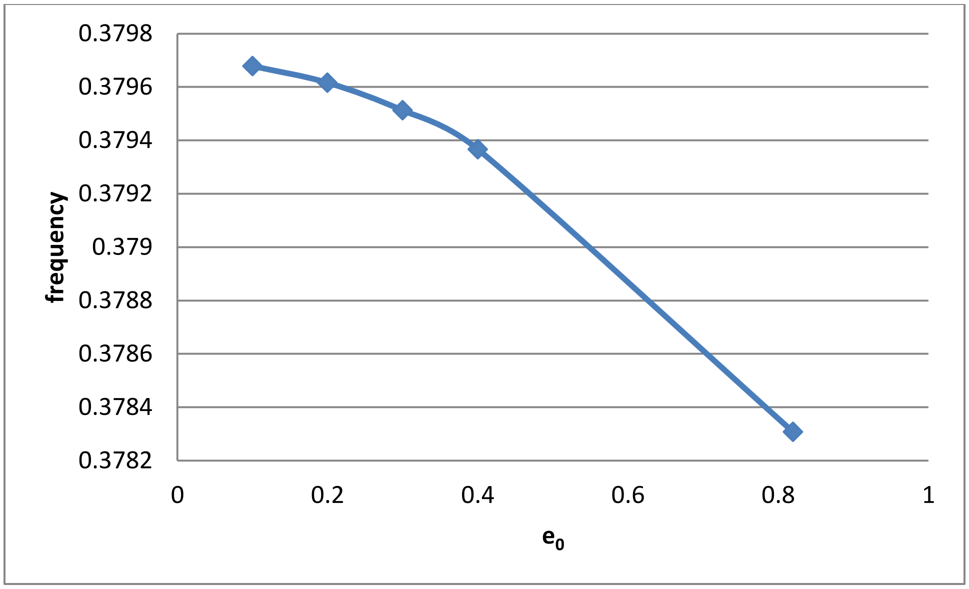

Fig 2: Effect of second frequency of non-local beam on various values of e_0.

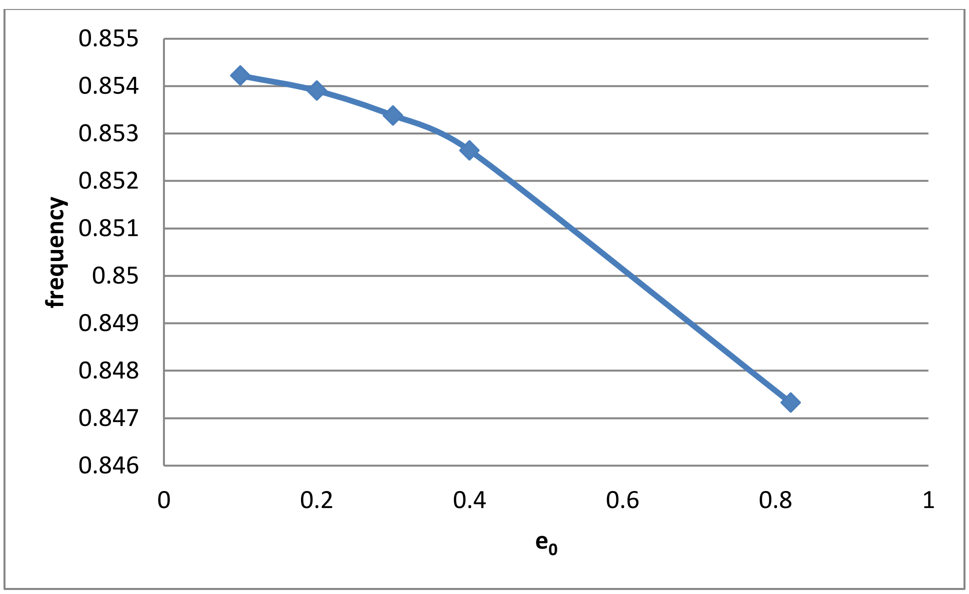

Fig 3: Effect of third frequency of non-local beam on various values of e_0.

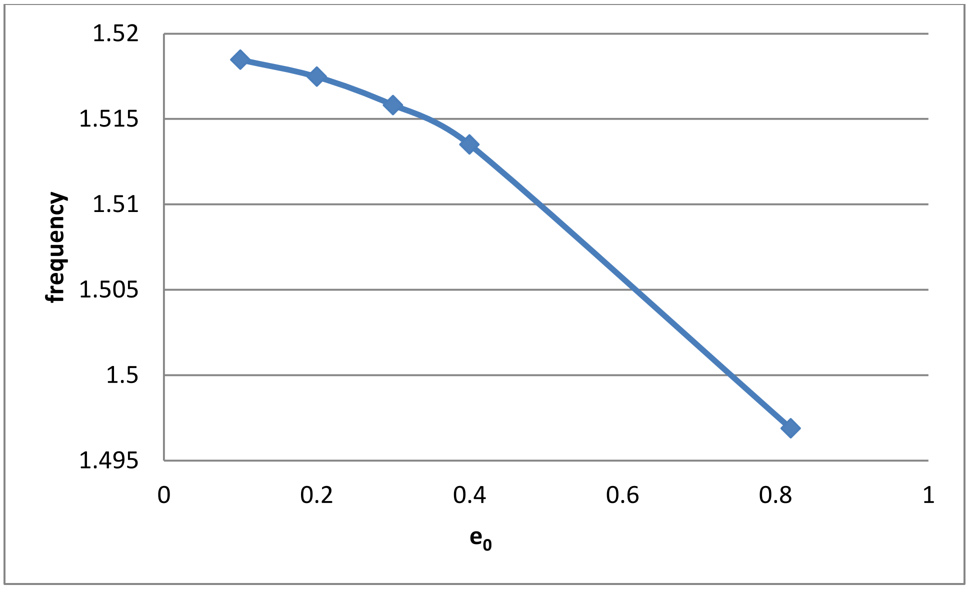

Fig 4: Effect of fourth frequency of non-local beam on various values of e_0.

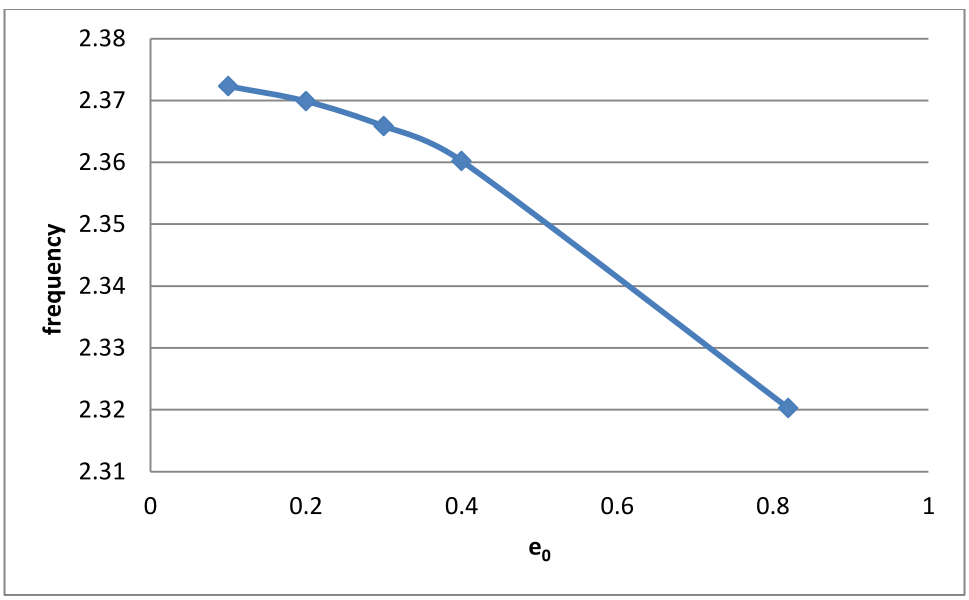

Fig 5: Effect of fifth frequency of non-local beam on various values of e_0

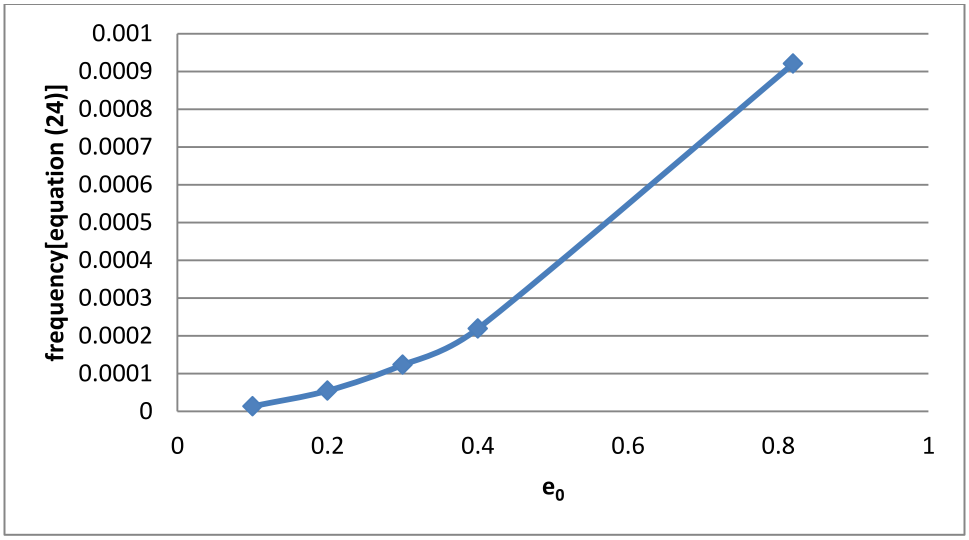

Fig 6: Impact of natural frequency of non-local beam on various values of e_0 for n=1 and L/a=60.

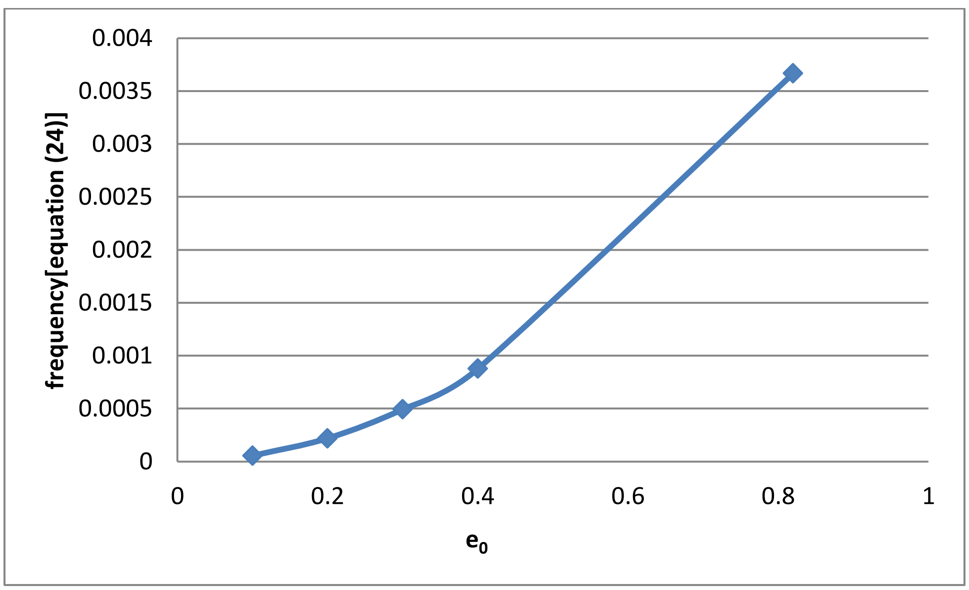

Fig 7: Impact of natural frequency of non-local beam on various values of e_0 for n=2 and L/a=60.

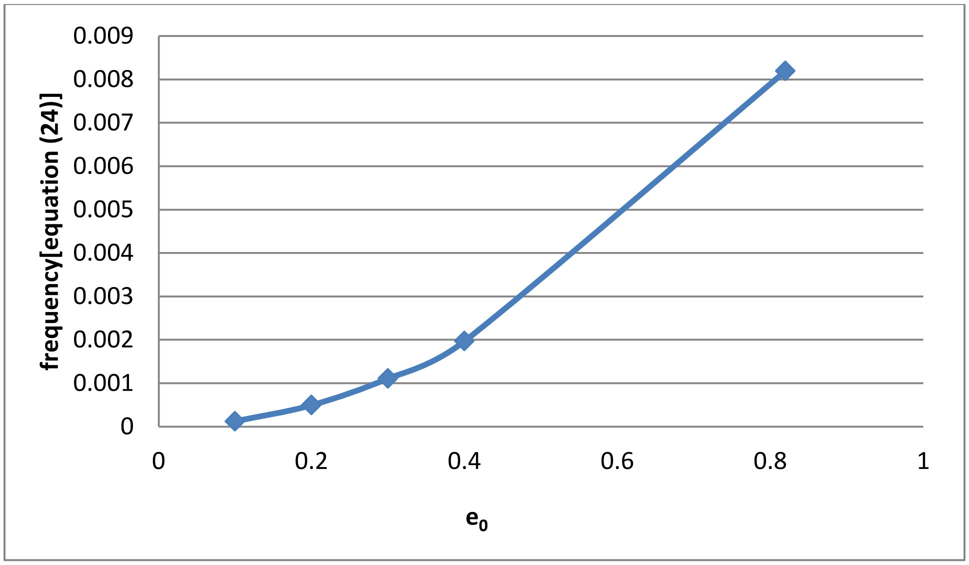

Fig 8: Impact of natural frequency of non-local beam on various values of e_0 for n=3 and L/a=60.

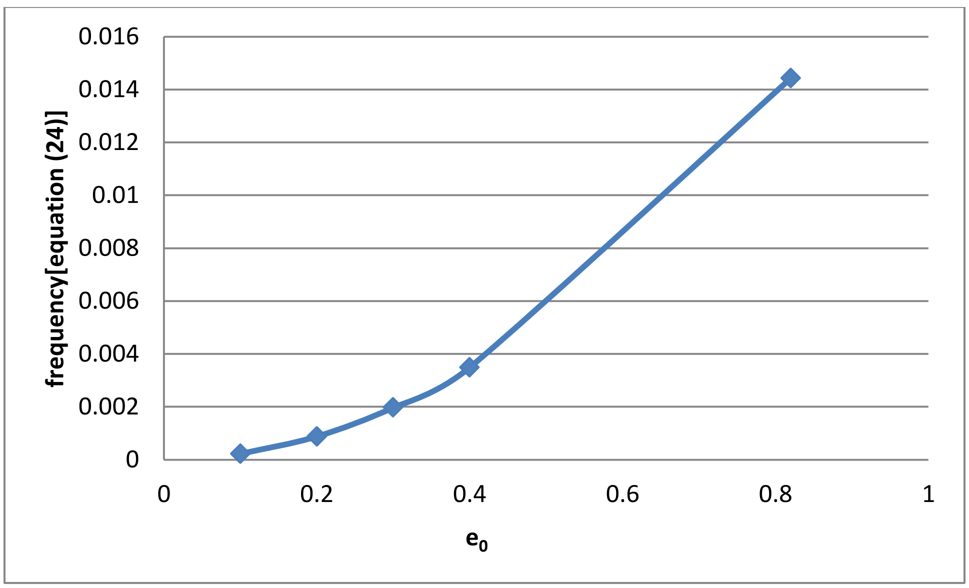

Fig 9: Impact of natural frequency of non-local beam on various values of e_0 for n=4 and L/a=60.

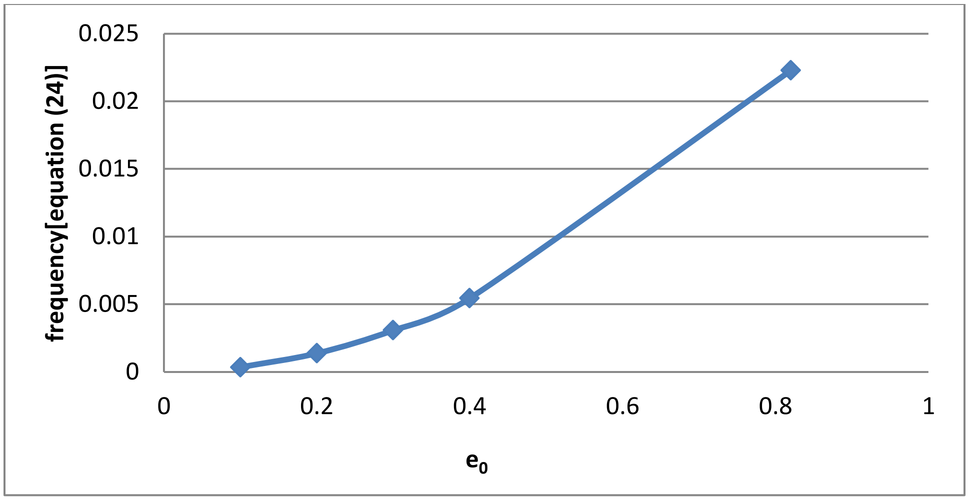

Fig 10: Impact of natural frequency of non-local beam on various values of e_0 for n=5 and L/a=60.

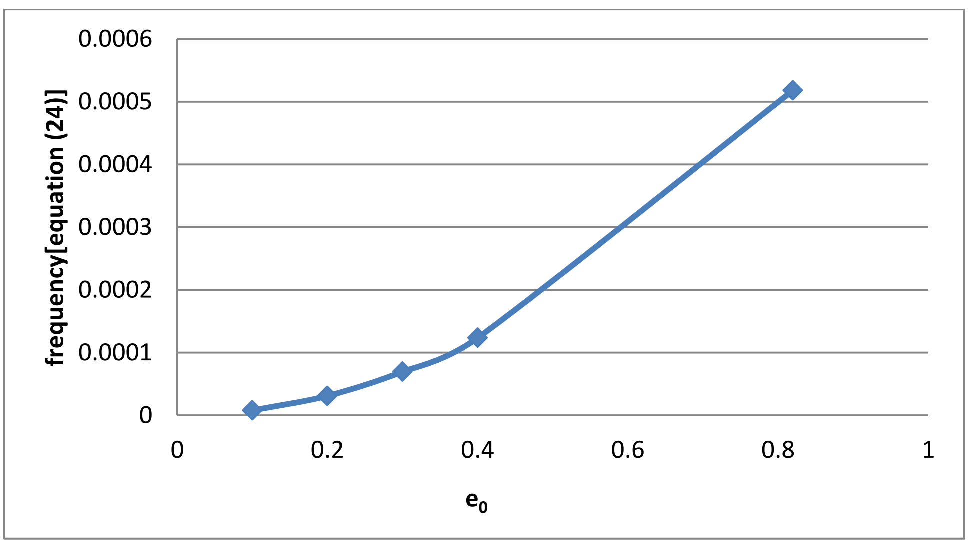

Fig 11: Impact of natural frequency of non-local beam on various values of e_0 for n=1 and L/a=80.

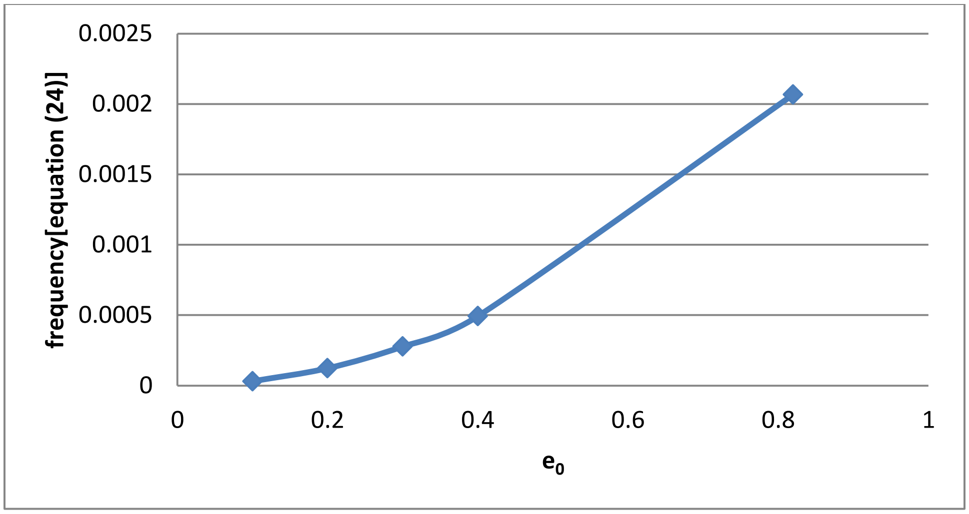

Fig 12: Impact of natural frequency of non-local beam on various values of e_0 for n=2 and L/a=80.

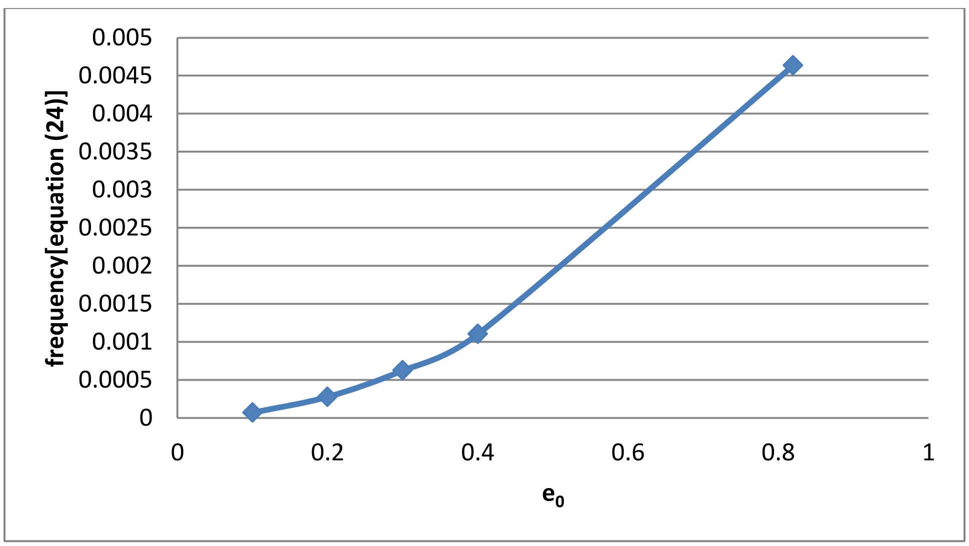

Fig 13: Impact of natural frequency of non-local beam on various values of e_0 for n=3 and L/a=80.

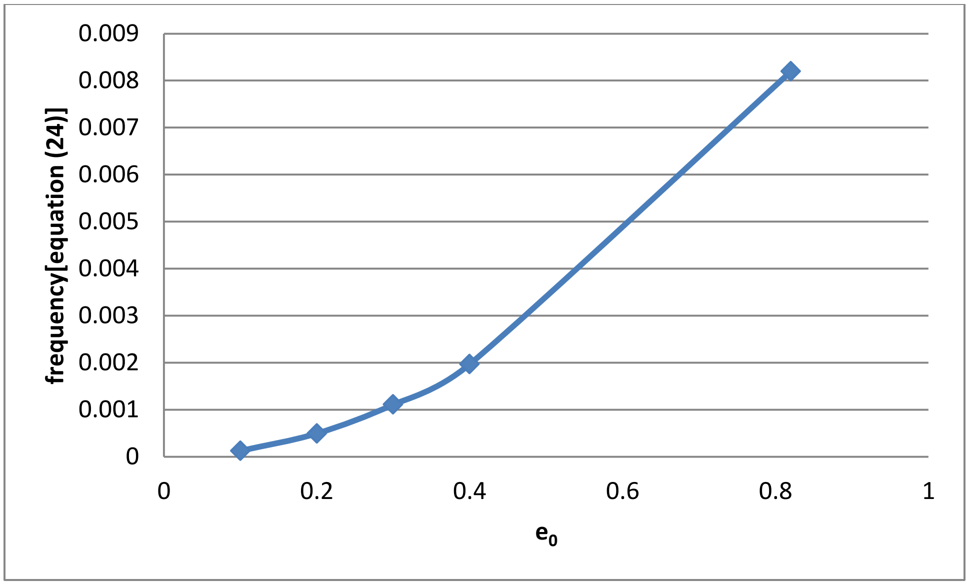

Fig 14: Impact of natural frequency of non-local beam on various values of e_0 for n=4 and L/a=80.

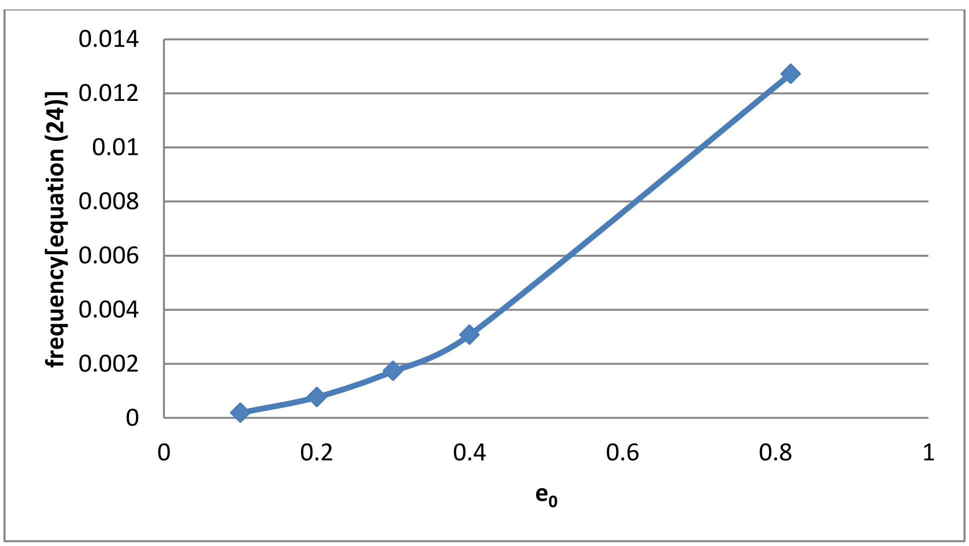

Fig 15: Impact of natural frequency of non-local beam on various values of e_0 for n=5 and L/a=80.

Chapter -5

Conclusion

Natural frequencies are obtained from the solution of the fourth order ordinary differential equation for nano-scale beam with simply supported boundary. From the result it reveals that the impact of non-local effect on the natural frequency increases with e_0. But the frequency decreases with increase in e_0. It may be seen that the frequency decreases with decrease in length for a particular e_0. Various parametric investigations as done may be useful in the nano-technology design. This investigation also gives a detail distinction of local and non-local Euler beam solutions and their results. The idea of the solution method may easily be extended to nano beams with other boundary conditions and complicating effects.

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List of Publications

1. Lipika Parida, Laxmi Behera, Analytical Solution and Parametric Study of Free Transverse Vibration of Nano Scale Beam, 2012, National Conference on Rough Set Theory and Artificial Intelligence by Department of Mathematics, Seemanta Mahavidyalaya.